The van Hiele Model and Learning Theories:
Implications for Teaching and Learning Geometry

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Introduction

The new Project Maths courses were examined in their entirety in the Junior and Leaving Certificate of 2015 and 2014 respectively. (Cosgrove, Perkins, Shiel, Fish and McGuinness (2012). These new courses represent a significant change in both subject content and pedagogy with teachers of mathematics attending ten full day workshops over a five year period. (PMDT, 2016a). "One of the key elements of Project Maths is a greater emphasis on an investigative approach, meaning that students become active participants in developing their mathematical knowledge and skills. This implies not only changes in the content of the syllabi, but also, and more fundamentally, perhaps, changes to teaching and learning approaches." (Cosgrove et al, 2012, p.7).

One of the five course 'strands' is 'Geometry and Trigonometry' which constitutes a substantial proportion (approximately 20%) of these courses. (PMDT, 2016b). In the first year of the Junior Certificate course, it is recommended that all schools follow a Common Introductory Course (CIC). (An Roinn Oideachais agus Scileanna, 2016). In order to help teachers implement this Common Introductory Course, the PMDT created a teacher handbook and online PDF. (PMDT, 2014 and PMDT 2011). Appendix A of this handbook is entitled "Geometry: Thinking at Different Levels" and describes how students learn geometry according to the van Hiele model (PMDT, 2014). The purpose of this paper is to examine how the van Hiele model, set in the context of two of the major learning theories, cognitivism and constructivism, can be used by teachers in the teaching and learning of geometry. It also compares and contrasts aspects of these learning theories with the van Hiele model. In particular, it provides a comparative analysis of the van Hiele model with the theoretical learning frameworks of Bloom, Gagne, Piaget, Vygotsky and Bruner. The concluding section of the paper discusses possible implications of the van Hiele model for eLearning design.

van Hiele Model (5 Levels)

The van Hiele model for the teaching and learning of geometry has its origins in separate doctoral dissertations by Pierre Marie van Hiele and his wife Dina van Hiele-Geldof submitted to the University of Utrecht in 1957. (van Hiele, 1957 and van Hiele-Geldof, 1957). Dina died soon after her thesis and it was left to her husband to explain and develop the theory in three papers written between 1958 and 1959. (Usiskin, 1982).

The van Hiele Model is based on five levels. (Usiskin, 1982). Hoffer (1979, 1981) describes the levels as follows: Level 0 (Recognition), Level 1 (Analysis), Level 2 (Order), Level 3 (Deduction), Level 4 (Rigor).

At Level 0, “the student can learn names of figures and recognizes a shape as a whole.” (Usiskin, 1982, p.4). “They will be able to distinguish shapes like triangles, squares, rectangles etc. but will not be able to explain, for example, what makes a rectangle a rectangle.” (PMDT, 2014, p.32). Students at this level see squares as being different to rectangles and not as a class of rectangle. (Hoffer, 1979, 1981).

At Level 1, “the student can identify properties of figures.” (Usiskin, 1982, p.4). “They understand that if a shape belongs to a class like “rectangle”, then it has all the properties of that class (two pairs of equal sides, right angles, two equal diagonals, two axes of symmetry)”. (PMDT, 2014, p.32). They understand that “rectangles have four right angles.” (Hoffer, 1979, 1981).

At Level 2, “the student can logically order figures and relationships.” (Usiskin, 1982, p.4). The objects of thought are the properties of shapes. They understand that a rectangle is a parallelogram since it has all the properties of a parallelogram as well as having all 90 degree angles. (PMDT, 2014, pp.32-33). “Simple deduction can be followed but proof is not understood.” (Hoffer, 1979, 1981).

At Level 3, “the student understands the significance of deduction and the roles of postulates, theorems, and proof." (Usiskin, 1982, p.4). The sequence of theorems “is arranged in such a manner that each theorem builds on the previous theorem(s).” (PMDT, 2014, p.33). “Proofs can be written with understanding.” (Hoffer, 1979, 1981).

At Level 4, “the student understands the necessity for rigor and is able to make abstract deductions.” (Usiskin, 1982, p.4). “Comparing different axiomatic systems is not done at secondary level.” (PMDT, 2014, p.33). “Non-Euclidean Geometry can be understood.” (Hoffer, 1979, 1981).

Burger and Shaughnessy (1986, p.31) conducted a research study of the van Hiele levels and found that the “students behavior on the tasks was consistent with the van Hieles’ original description of the levels, although the discreteness of levels, particularly of analysis and abstraction, was not confirmed.”

van Hiele Model (5 Phases)

van Hiele believed that the learning process leading to complete understanding at the next higher level has five phases, approximately but not strictly sequential, entitled: “inquiry, directed orientation, explanation, free orientation, integration.” (Usiskin, 1982, p.6). Gutierrez (2007) published an excellent online interactive mind map of the van Hiele theory. Both Usiskin (1982) and Gutierrez (2007) have described these phases (below) while Howse and Howse (2014) provided a summary of them within parentheses:

Phase 1- Inquiry/Information: (Students develop vocabulary and concepts for a particular task)
Phase 2 - Directed Orientation: (Students actively engage in teacher-directed tasks)
Phase 3 - Explication: (Students are given the opportunity to verbalise their understanding)
Phase 4 - Free Orientation: (Students are challenged with discovering their own ways of completing each task)
Phase 5 - Integration: (Students summarise what they have learned).

van Hiele Levels and Gagne's and Bloom's Hierarchies

Yazdani (2008, p.58) notes that "There are striking similarities between the van Hieles' model of levels of understanding geometry and the hierarchical levels of learning developed by Robert Gagne." This taxonomy of learning outcomes consists of five categories of learning outcomes - verbal information, intellectual skills, cognitive strategies, attitudes, and motor skills. (Gagné, Briggs and Wager, 1992). Bloom, Engelhart, Furst, Hill, and Krathwohl (1956) identified a hierarchy of six categories of cognitive skills: knowledge, comprehension, application, analysis, synthesis and evaluation. This hierarchy of skills is similar to the van Hiele hierarchy of levels insofar as both hierarchies represent a progression from lower order to higher order skills. Their main difference is that van Hiele believed it was necessary to go through each level to reach the next level. (Usiskin, 1982). However, a revised version of Bloom's taxonomy held the view that only the three lowest levels of Bloom's taxonomy are hierarchically ordered while the three higher levels are parallel. (Anderson and Krathwohl, 2001).

Gagné et al (1992, p.44) list "verbal information" as one of the "five kinds of learned capabilities" and I think this can perhaps be mapped to van Hiele's Level 0 where "the student can learn names of figures…" (Usiskin, 1982, p.4). Similarly, the capability of identifying the diagonal of a rectangle is provided as an example of the capability "intellectual skill" by Gagné et al (1992, p.44) which appears to have it's equivalent in van Hiele's Level 1 where students can understand that "rectangles have four right angles" (Hoffer, 1979, 1981) or that a rectangle "has two equal diagonals" (PMDT, 2014, p.32). A research project to investigate the effectiveness of the levels in the van Hiele model versus one of Gagné's hierarchies (8 levels) on learning geometry in a P-8 pre-service teacher's environment found that there was no significant difference between the two theoretical learning models on learning geometry. (Yazdani, 2008). However, this research project compared the five van Hiele levels with the the eight different classes of learning outlined in his earlier work (Gagné, 1965) rather than one of his later collaborative works (Gagné et al, 1992).

van Hiele Levels and Piaget's Stages

Piaget (1953) argues that children do not enter the formal operational stage until they are 14 years of age and that they cannot learn formal proofs before this period. van Hiele (1985) describes similar properties in his penultimate geometric level deduction although he does not specify which age pupils reach this level. In Irish secondary schools, students generally don't study formal proofs in geometry until second or third year when they are approximately 14 years old. Even then, only students following the higher level course for examination at Junior Certificate are required to learn formal proofs. (An Roinn Oideachais ages Scileanna, 2016).

"Piaget's theory suggests that children need to develop a specific cognitive structure before they will be able to perform such tasks as problem solving or abstract thinking." (Yazdani, 2008, p.58). However, according to Usiskin (1982), van Hiele differs to Piaget in believing that cognitive development in geometry can be accelerated by instruction. This is consistent with Bruner's philosophy that a child can learn any task given the right teaching. (Bruner, 1960).

An approach encapsulating the developmental stage theories of Jean Piaget would dictate a cognitive development approach based on the student's biological maturation and environmental experience. For example, at the concrete operational stage (third stage, 7 to 11 years old) Piaget hypothesised that a child can only solve problems that apply to concrete events or objects whereas at the formal operational stage (fourth stage, 11 to 15-20 years old), a child has developed the ability to think about abstract concepts and can demonstrate deductive reasoning, in which they draw specific conclusions from abstract concepts using logic. (Piaget, 1953).

van Hiele Phases and the work of Bruner and Vygotsky

Vygotsky (1978) makes the case for a social constructivist model of learning where the student learns from More Knowledgeable Others (MKOs) and their peers. He claimed that learning occurs when children are tackled with a task that they cannot accomplish alone but which they can do when given help by a peer or a teacher. He called this immediate level of development, above one's present level of development, the "zone of proximal development." An important application of Vygosky's theory is the idea of helping the learners with the concepts and skills above their zone of proximal development to guide them to self-discovery. This process is called "scaffolding". (Yazdani, 2008). Wood, Bruner, and Ross (1976, p.90) introduced the term and defined it as a process "that enables a child or novice to solve a task or achieve a goal that would be beyond his unassisted efforts." Vygotsky's scaffolding appears to be quite similar to van Hiele's phases.

The first two phases of the van Hiele model are 'Inquiry' where questions are asked and 'Directed Orientation' where students explore the topic through materials that the teacher has carefully sequenced. (Usiskin, 1982; Gutierrez, 2007). Bruner (1961) is often credited with originating discovery learning where the student is not provided with an exact answer but instead is given the materials to find the answer themselves. He hypothesised that embedding information in a cognitive structure that a person has constructed would make the information more accessible for retrieval from memory.

Just as Piaget viewed development as progressing from the physical experience of the child to representative schema which facilitate mental operations, Bruner also distinguished between three modes of representation or systems of processing in both the physical and mental realm. He referred to these three systems of processing as enactive (actions), iconic (images), and symbolic (words and symbols). Like Piaget, Bruner hypothesised that these representations occurred at particular stages (and ages) of a child's development. (Bruner, 1964). Van Hiele offered no developmental timetable for growth through the levels. Instead, in ways that have much in common with Vygotsky (1978), he saw development in terms of students' interaction with the cultural environment, their own exploration, and their reaction to a guided learning process. (van Hiele, 1986).

Implications for eLearning and Instructional Design

Training courses were held in Education Centres throughout Ireland in the 2010/11 school year for teachers interested in using Excel (spreadsheet application) and GeoGebra (dynamic geometry application) in the teaching and learning of Mathematics. (PMDT, 2010). The PMDT state that "Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package." (PMDT, 2014, p.4). GeoGebra's design and functionality is consistent with a constructivist approach to learning. (Kul, 2013).

The van Hiele model and the developmental stage theories of Jean Piaget provide eLearning instructional designers with a knowledge of what a student can and can't understand at various levels or stages of learning. Gagné's taxonomy of learning outcomes and Bloom's taxonomy of educational objectives provide for different modes of assessing whether learning has occurred at the different van Hiele levels. (Usiskin, 1982; Gagné et al, 1992; Bloom et al, 1956). Perhaps this knowledge could guide the instructional design process in the development of eLearning courses for geometry.

The van Hiele phases of learning within each level, as well as the levels themselves, should instruct the eLearning development team (instructional designer, subject matter expert, course authoring specialist, etc) as to the scope and limitations of behaviourist, cognitive and constructivist approaches to teaching geometry at the various levels. (Clark and Mayer, 2011). Since van Hiele hypothesises that the learner cannot go to to the next level without passing through the preceding level, this has important implications for the instructional design of a future personalised geometry learning system. (Usiskin, 1982; Clark and Mayer, 2011).

References

An Roinn Oideachais agus Scileanna (2016). Junior Certificate syllabus for examination from 2016. Retrieved September 25, 2016, from http://www.curriculumonline.ie/getmedia/4f6cba68-ac41-485c-85a0-32ae6c3559a7/JCSEC18_Maths_Examination-in-2016.pdf

Anderson, L.W. & Krathwohl, D.R. (Eds.). (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives*. New York: Longman.

Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H. & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: The classification of educational goals*. *Handbook I: Cognitive domain*. New York: Longman.

Bruner, J.S. (1961). The act of discovery. *Harvard Educational Review*, 31 (1): 21–32.

Bruner, J.S. (1964). The course of cognitive growth. *American Psychologist*, 19 (1): 1-15.

Bruner, J.S. (1960). *The Process of Education*. Cambridge, Mass.: Harvard University Press.

Burger, W.F. & Shaughnessy, M.J. (1986). Characterizing the van Hiele Levels of Development in Geometry. *Journal for Research in Mathematics Education*, 17(1), 31-48.

Clark, R.C. & Mayer, R.E. (2011). *E-Learning and the Science of Instruction: Proven Guidelines for Consumers and Designers of Multimedia Learning*. San Francisco, CA: Pfeiffer.

Cosgrove, J., Perkins, R., Shiel, G., Fish, R., & McGuinness, L. (2012). *Teaching and Learning in Project Maths: Insights from Teachers who Participated in PISA 2012*. Dublin: Educational Research Centre, St. Patrick's College.

Gagné, R. M., Briggs, L. J., & Wager, W. W. (1992). *Principles of Instructional Design*. Fort Worth: Harcourt Brace Jovanovic.

Gagne, R. (1965). *The conditions of learning*. New York: Holt, Rinehart and Winston, Inc.

Gutierrez, A. (2007). Hiele Model of Geometric Thought. *Geometry Step by Step From the Land of the Incas*. Retrieved October 10th, 2016, from http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Hoffer, A. (1979). *Geometry. Teachers Edition*. Menlo Park, CA: Addison -Wesley.

Hoffer, A. (1981). Geometry is More than Proof. *The Mathematics Teacher*, 74, 11-18.

House, T.D. & Howse, M.E. (2014). Linking the Van Hiele Theory to Instruction. *National Council of Teachers of Mathematics*, 21(5). Retrieved October 1, 2016, from http://www.nctm.org/Publications/teaching-children-mathematics/2014/Vol21/Issue5/Linking-the-Van-Hiele-Theory-to-Instruction

Kul, U. (2013). Professional Development of Turkish Primary Mathematics Teachers within a Computer-Integrated Learning Environment: An Exploration of Changes of Beliefs. (Unpublished doctoral dissertation). University of Leicester.

Piaget, J. (1953). *The Origin of Intelligence in the Child*. London: Routledge and Kegan Paul.

Project Maths Development Team (PMDT) (2010). Courses. Retrieved October 15, 2016, from http://ict.projectmaths.ie/courses

Project Maths Development Team (PMDT). (2011). *Geometry: Van Hiele Model of Geometric Thought: Thinking at Different Levels*. Retrieved October 2, 2016, from www.projectmaths.ie/workshops/workshop2/VanHiele.pdf

Project Maths Development Team (PMDT) (2014). Teacher Handbook First Year. Retrieved September 24, 2016, from http://www.projectmaths.ie/documents/handbooks/firstyearhandbook2015.pdf

Project Maths Development Team (PMDT) (2016a). Workshops & Seminar. Retrieved September 24, 2016, from http://www.projectmaths.ie/for-teachers/workshops

Project Maths Development Team (PMDT) (2016b). Project Maths Frequently Asked Questions. Retrieved September 25, 2016, from http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Project_Maths_Frequently_Asked_Questions/Project_Maths_Frequently_Asked_Questions.html

Usiskin, Z (1982). Van Hiele Levels and Achievement in Secondary School Geometry. Retrieved September 24, 2016, from http://ucsmp.uchicago.edu/resources/van_hiele_levels.pdf van Hiele, P.M. (1957). *The problem of insight, in connection with schoolchildren's insight into the subject of geometry*. English summary (by P.M. van Hiele) of De Problematiek van het Inzicht Gedemonstreed van het Inzicht von Schoolkindren in Meetkundeleerstof (Unpublished doctoral dissertation). University of Utrecht.

van Hiele-Geldof, D. (1957). The didactics of geometry in the lowest class of the secondary school. English summary by Dina van Hiele-Geldof) of De didaktiek van de Meetkunde in de eerste klass van het V.H.M.O. (Unpublished doctoral dissertation). University of Utrecht.

van Hiele, P. (1985). *The Child's Thought and Geometry*. Brooklyn, NY: City University of New York.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wood, D., Bruner, J.S. & Ross, G. (1976). The Role of the Tutor in Problem Solving. *Journal of Child Psychology and Psychiatry*. 17: 89-100. Nottingham, Oxford and Harvard Universities: Pergamon Press. Retrieved October 7, 2016, from http://onlinelibrary.wiley.com/doi/10.1111/j.1469-7610.1976.tb00381.x/epdf

Yazdani, M. A. (2008). The Gagne – van Hieles Connection: A Comparative Analysis of Two Theoretical Learning Frameworks. *Journal of Mathematical Sciences & Mathematics Education*, 3(1), 58-63.